APPENDIX G ESTABLISHING MQOS FOR MEASUREMENT UNCERTAINTY, MDCs AND MQCs

G.1 Establishing MQOs

This section provides the rationale and guidance for establishing project-specific MQOs for controlling σ_M. This control is achieved by establishing a desired maximum measurement method uncertainty at the upper boundary of the gray region. This control also will assist in both the measurement method selection process and in the evaluation of measurement data. Approaches applicable to several situations are detailed below.

Table G.1 Notation for Section G.1

Symbol	***Definition***	***Formula or reference***	***Type***
α	Probability of a Type I decision error		Chosen during DQO process
β	The probability of a Type II decision error		Chosen during DQO process
Δ	Width of the gray region	(UBGR-LBGR)	Chosen during DQO process
φ_{MR}	Required relative method uncertainty above the UBGR	u_{MR} / UBGR	Chosen during DQO process
S_{C}.	The critical value of the net instrument signal (e.g., net count)	Calculation of S_{C} requires the choice of a significance level for the test. The significance level is a specified upper bound for the probability, α, of a Type I error. The significance level is usually chosen to be 0.05.	If a measured value exceeds the critical value, a decision is made that radiation or radioactivity has been detected
σ	The total standard deviation of the data	$(\sigma_S^2 + \sigma_M^2)^{½}$	Theoretical population parameter
σ_S	Standard deviation of the concentration in the sampled population		Theoretical population parameter
σ_M	Standard deviation of the measurement method		Theoretical population parameter
u_{MR}	Required method uncertainty at and below the UBGR	Upper bound to the value of σ_M	Chosen during DQO process
$u_c^2(y)$	Combined variance of y	Uncertainty propagation	
$u_{\mathrm{c}}(y)$	Combined standard uncertainty of y.	Uncertainty propagation	
$z_{1-\alpha}$ ($z_{1-\beta}$)	1-α (or 1-β) quantile of a standard normal distribution function	Table of Standard normal distribution.	Theoretical

G.1.1 Developing a Requirement for Measurement Method Uncertainty For MARSSIM-Type surveys

When, as in MARSSIM-Type surveys, a decision is to be made about the mean of a sampled population, generally the average of a set of measurements on a survey unit is compared to the disposition criterion.

The total variance of the data, σ^2, is the sum of two components

$$\sigma^2 = \sigma_M^2 + \sigma_S^2 \tag{G-1}$$

Where:

σ_M^2 = measurement method variance (M = “measurement”), and

σ_S^2 = variance of the radionuclide concentration or activity concentration in the sampled population (S = “sampling”).

The spatial and temporal distribution of the concentration, the extent of the survey unit, the physical sizes of the measured material, and the choice of measurement locations may affect the sampling standard deviation, σ_S. The measurement standard deviation, σ_M , is affected by the measurement methods. The value of σ_M is estimated in MARSAME by the combined standard uncertainty of a measured value for a measurement of material whose concentration equals the hypothesized population mean concentration. The calculation of measurement uncertainties is covered in Section 5.6.

Four cases are considered below where target values for σ_M can be suggested depending on what is known about σ_S. Cases 1 and 2 treat the desired overall objective of keeping $\Delta/\sigma \approx 3$ or higher. When this is not possible, Cases 3 and 4 treat the less desirable alternative of attempting to prevent Δ/σ from going lower than 1.

Case 1: σ_S is known relative to Δ / 3

Generally, it is easier to control σ_M than σ_S. If σ_S is known (approximately), a target value for σ_M can be determined.

Case 1a: $\sigma_S \leq \Delta / 3$

If $\sigma_S \leq \Delta / 3$, then a value of σ_M no greater than $\sqrt{(\Delta^2 / 9) - \sigma_S^2}$ ensures that $\sigma \leq \Delta / 3$, because we have $\sigma^2 = \sigma_M^2 + \sigma_S^2 \leq (\Delta^2 / 9 - \sigma_S^2) + \sigma_S^2 = \Delta^2 / 9$, as desired.

Case 1b: $\sigma_S > \Delta / 3$

If $\sigma_S > \Delta / 3$, the requirement that the total σ be less than $\Delta/3$ cannot be met regardless of σ_M. In this case, it is sufficient to make σ_M negligible in comparison to σ_S. Generally, σ_M can be considered negligible in comparison to σ_S if it is no greater than $\sigma_S/3$.

Case 2: σ_S is not known relative to $\Delta / 3$

Often one needs a method for choosing σ_M in the absence of specific information about σ_S. Since it is desirable to have $\sigma \leq \Delta / 3$, this condition is adopted as a primary requirement. Assume for the moment that σ_S is large. Then σ_M should be made negligible by comparison. As mentioned above, σ_M can be considered negligible if it is no greater than $\sigma_S/3$. When this condition is met, further reduction of σ_M has little effect on σ and therefore is usually not cost-effective. So, the inequality $\sigma_M \leq \sigma_S/3$ is adopted as a secondary requirement.

Starting with the definition $\sigma^2 = \sigma_M^2 + \sigma_S^2$ and substituting the secondary requirement $\sigma_M \leq \sigma_S/3$ we get $\sigma^2 \geq \sigma_M^2 + 9\sigma_M^2 = 10\sigma_M^2$, thus

$$\sigma_M \leq \frac{\sigma}{\sqrt{10}} \qquad \textbf{(G-2)}$$

Substituting the primary requirement that $\Delta/\sigma \geq 3$ (i.e., $\sigma \leq \Delta / 3$) we get $\sigma_M \leq \frac{\sigma}{\sqrt{10}} \leq \frac{\Delta / 3}{\sqrt{10}}$, thus

$$\sigma_M \leq \frac{\Delta}{3\sqrt{10}} \qquad \textbf{(G-3)}$$

Or approximately

$$\sigma_M \leq \frac{\Delta}{10} \qquad \textbf{(G-4)}$$

The required upper bound for the standard deviation σ_M will be denoted by σ_{MR}. MARSAME recommends the equation

$$\sigma_{MR} = \frac{\Delta}{10} \qquad \textbf{(G-5)}$$

by default as a requirement when σ_S is unknown and a decision is to be made about the mean of a sampled population.

This upper bound was derived from the assumption that σ_S was large, but it also ensures that the primary requirement $\sigma \le \Delta / 3$ (i.e., $\Delta / \sigma \ge 3$) will be met if σ_S is small. When the measurement standard deviation σ_M is less than σ_{MR}, the primary requirement will be met unless the sampling variance, σ_S^2, is so large that σ_M^2 is negligible by comparison, in which case little benefit can be obtained from further reduction of σ_M.

It may be that the primary requirement that Δ/σ be at least 3 is not achievable. Suppose that the primary requirement is relaxed to achieving Δ/σ at least 1 (i.e., $\sigma \le \Delta$). This leads to consideration of:

Case 3: σ_S is known relative to Δ

As in Case 1, it is generally easier to control σ_M than σ_S. If σ_S is known (approximately), a target value for σ_M can be determined.

Case 3a: $\sigma_S \le \Delta$

If $\sigma_S \le \Delta$, then a value of σ_M no greater than $\sqrt{\Delta^2 - \sigma_S^2}$ ensures that $\sigma \le \Delta$, because we have $\sigma^2 = \sigma_M^2 + \sigma_S^2 \le (\Delta^2 - \sigma_S^2) + \sigma_S^2 = \Delta^2$ as desired.

Case 3b: $\sigma_S > \Delta$

If $\sigma_S > \Delta$, the requirement that the total σ be less than Δ cannot be met regardless of σ_M. In this case, it is sufficient to make σ_M negligible in comparison to σ_S. Generally, σ_M can be considered negligible if it is no greater than $\sigma_S/3$.

Case 4: σ_S is not known relative to Δ

Suppose $\sigma \le \Delta$ is adopted as the primary requirement. As in Case 2, if σ_S is large then σ_M should be made negligible by comparison. As mentioned above, σ_M can be considered negligible if it is no greater than $\sigma_S/3$. When this condition is met, further reduction of σ_M has little effect on σ and therefore is usually not cost-effective. So, the inequality $\sigma_M \le \sigma_S/3$ is adopted as a secondary requirement.

Starting with the definition $\sigma^2 = \sigma_M^2 + \sigma_S^2$ and substituting the secondary requirement $\sigma_M \le \sigma_S / 3$ we get $\sigma^2 \ge \sigma_M^2 + 9\sigma_M^2 = 10\sigma_M^2$, thus

$$\sigma_M \le \frac{\sigma}{\sqrt{10}}$$

Substituting the primary requirement that $\Delta/\sigma \ge 1$ (i.e., $\sigma \le \Delta$) we get $\sigma_M \le \frac{\sigma}{\sqrt{10}} \le \frac{\Delta}{\sqrt{10}}$, thus

$$\sigma_M \le \frac{\Delta}{\sqrt{10}} \approx \frac{\Delta}{3}$$

G.1.2 Developing a Requirement for Measurement Method Uncertainty When Decisions Are to Be Made About Individual Items

When decisions are to be made about individual items, the total variance of the data equals the measurement variance, σ_M^2, and the data distribution in most instances should be approximately normal. The decision in this case may be made by comparing the measured concentration, x, plus or minus a multiple of its combined standard uncertainty, to the action level. The combined standard uncertainty, $u_c(x)$, is assumed to be an estimate of the true standard deviation of the measurement process as applied to the item being measured; so, the multiplier of $u_c(x)$ equals $z_{1-\alpha}$, the $(1-\alpha)$-quantile of the standard normal distribution (see MARLAP appendix C).

Alternatively, if AL = 0, so that any detectable amount of radioactivity is of concern, the decision may involve comparing the net instrument signal (e.g., count rate) to the critical value of the concentration, S_C, as defined in Section 5.7.1.

Two cases are considered below where target values for σ_M can be suggested depending on what is known about the width of the gray region and the desired Type I and Type II decision error rates. Case 5 is for Scenario A, and Case 6 is for Scenario B.

Case 5: Suppose the null hypothesis is $X \geq$ AL (see Scenario A in Chapter 4), so that the action level is the upper bound of the gray region. Given the measurement variance σ_M^2, only a measured result that is less than (UBGR – $z_{1-\alpha}\sigma_M$) will be judged to be clearly less than the action level. Then the desired power of the test $1 - \beta$ is achieved at the lower bound of the gray region only if the LBGR ≤ UBGR – $z_{1-\alpha}\sigma_M - z_{1-\beta}\sigma_M$. Algebraic manipulation transforms this requirement to

$$\sigma_M \leq \frac{\text{UBGR - LBGR}}{z_{1-\alpha} + z_{1-\beta}} = \frac{\Delta}{z_{1-\alpha} + z_{1-\beta}} \qquad \textbf{(G-6)}$$

Case 6: Suppose the null hypothesis is $X \leq$ AL (see Scenario B in Chapter 4), so that the action level is the lower bound of the gray region. In this case, only a measured result that is greater than LBGR + $z_{1-\alpha}\sigma_M$ will be judged to be clearly greater than the action level. The desired power of the test $1 - \beta$ is achieved at the upper bound of the gray region only if the UBGR ≥ LBGR + $z_{1-\alpha}\sigma_M + z_{1-\beta}\sigma_M$. Algebraic manipulation transforms this requirement to:

$$\sigma_M \leq \frac{\text{UBGR - LBGR}}{z_{1-\alpha} + z_{1-\beta}} = \frac{\Delta}{z_{1-\alpha} + z_{1-\beta}}$$

So, in either Scenario A or Scenario B, the requirement remains that:

$$\sigma_M \leq \frac{\Delta}{z_{1-\alpha} + z_{1-\beta}} \qquad \textbf{(G-7)}$$

Therefore, MARSAME uses the equation:

$$u_{MR} = \sigma_{MR} = \frac{\Delta}{z_{1-\alpha} + z_{1-\beta}} \qquad \textbf{(G-8)}$$

as an MQO for method uncertainty when decisions are to be made about individual items or locations and not about population parameters.

If both α and β are at least 0.05, one may use the value $u_{MR} = 0.3\Delta$.

The recommended value of u_{MR} is based on the assumption that any known bias in the measurement process has been corrected and that any remaining bias is well less than a third of the method uncertainty.

G.2 Uncertainty Calculation

Table G.2 Notation for Section G.2

Symbol	***Definition***	***Formula or reference***	***Type***
a	Half-width of a bounded probability distribution	Type B evaluation of uncertainty	Estimated
c_i	Sensitivity coefficient	$\partial f / \partial x_i$, the partial derivative of f with respect to x_i	Evaluated at the measured values $x_1, x_2, \ldots, x_N$
$f(x_1, x_2, \ldots, x_N)$	The calculated value of the output quantity from measurable input quantities for a particular measurement	$y = f(x_1, x_2, \ldots, x_N)$	Experimental
$f(X_1, X_2, \ldots, X_N)$	Model equation expressing the mathematical relationship, between the measurand, Y and the input quantities X_i.	$Y = f(X_1, X_2, \ldots, X_N)$	Theoretical
k	Coverage factor for expanded uncertainty	Numerical factor used as a multiplier of the combined standard uncertainty in order to obtain an expanded uncertainty	Chosen during DQO process
p	Coverage probability for expanded uncertainty	Probability that the interval surrounding the result of a measurement determined by the expanded uncertainty will contain the value of the measurand	Chosen during DQO process
$r(x_i, x_j)$	Correlation coefficient for two input estimates, x_i and x_j,	$u(x_i, x_j) / (u(x_i)\, u(x_j))$	Experimental
$s(x_i)$	Sample standard deviation of the input estimate x_i	$s(x_i) = \sqrt{\frac{1}{(n-1)} \sum_{k=1}^{n} (x_{i,k} - \overline{x}_i)^2}$	Experimental
$u(x_i)$	Type B standard uncertainty of the input estimate x_i		Estimated
$u_i(y)$	Component of the combined standard uncertainty $u_c(y)$ generated by the standard uncertainty of the input estimate x_i, $u(x_i)$	$u_i(y) = c_i u(x_i)$	Estimated
$u_c(y)$	Combined standard uncertainty of y.	Uncertainty propagation	
$u_c^2(y)$	Combined variance of y	Uncertainty propagation	

Table G.2 Notation for Section G.2 (continued)

Symbol	*Definition*	*Formula or reference*	*Type*
U	Expanded uncertainty	"Defining an interval about the result of a measurement that may be expected to encompass a large fraction of values that could reasonably be attributed to the measurand" (GUM)	
$u(x_i,x_j)$	Covariance of two input estimates, x_i and x_j,		Experimental
$u_c(y)/y$	Relative combined standard uncertainty of the output quantity for a particular measurement		Experimental
$u(x_i)/x_i$	Relative standard uncertainty of a nonzero input estimate x_i for a particular measurement		Experimental
$w_1, w_2, \ldots, w_n$	input quantities appearing in the numerator of $y = f(x_1, x_2, \ldots, x_N)$	See $z_1, z_2, \ldots, z_m$ below	
$X_1, X_2, \ldots, X_N$	Measurable input quantities		Theoretical
$x_1, x_2, \ldots, x_N$	Estimates of the measurable input quantities for a particular measurement		Experimental
Y	The output quantity or measurand		Theoretical
y	Estimate of the output quantity for a particular measurement		Experimental
$z_1, z_2, \ldots, z_m$	input quantities appearing in the denominator of $y = f(x_1, x_2, \ldots, x_N)$	$N=n+m$	

G.2.1 Procedures for Evaluating Uncertainty

The usual eight steps for evaluating and reporting the uncertainty of a measurement are summarized in the following subsections (adapted from Chapter 8 of the GUM):

G.2.1.1 Identify the Measurand, Y, and all the Input Quantities, X_i, for the Mathematical Model

Include all quantities whose variability or uncertainty could have a potentially significant effect on the result. Express the mathematical relationship, $Y = f(X_1, X_2, \ldots, X_N)$, between the measurand and the input quantities.

The procedure for assessing the uncertainty of a measurement begins with listing all significant sources of uncertainty in the measurement process. A good place to begin is with the input quantities' mathematical model $Y = f(X_1, X_2, \ldots, X_N)$. When an effect in the measurement process that is not explicitly represented by an input quantity has been identified and quantified, an additional quantity should be included in the mathematical measurement model to correct for it. The quantity, called a correction (additive with a nominal value of zero) or correction factor (multiplicative with a nominal value of one), will have an uncertainty that should also be evaluated and propagated. Each uncertainty that is potentially significant should be evaluated quantitatively.

G.2.1.2 Determine an Estimate, x_i, of the Value of Each Input Quantity, X_i

This involves simply determining for the particular measurement at hand, the specific value, x_i, that should be substituted for the input quantity X_i in the mathematical relationship, $Y = f(X_1, X_2, \ldots, X_N)$.

G.2.1.3 Evaluate the Standard Uncertainty, $u(x_i)$, for Each Input Estimate, x_i, Using a Type A Method, a Type B Method, or a Combination of Both

Methods for evaluating standard uncertainties are classified as either "Type A" or "Type B" (NIST, 1994). Both types of uncertainty need to be taken into consideration. A Type A evaluation of an uncertainty uses a series of measurements to estimate the standard deviation empirically. Any other method of evaluating an uncertainty is a Type B method. A Type B evaluation of standard uncertainty is usually based on scientific judgment using all the relevant information available, which may include:

- Previous measurement data,
- Experience with, or general knowledge of, the behavior and property of relevant materials and instruments,
- Manufacturer's specifications,
- Data provided in calibration and other reports, and
- Uncertainties assigned to reference data taken from handbooks.

The Type A standard uncertainty of the input estimate x_i is defined to be the experimental standard deviation of the mean:

$$u(x_i) = \sqrt{\frac{1}{n(n-1)}\sum_{k=1}^{n}(x_{i,k} - \overline{x}_i)^2} = s(x_i)/\sqrt{n} \qquad \textbf{(G-9)}$$

Example 1: Type A uncertainty calculation using equation G-9**:**

Ten independent one-minute measurements of the counts from a check source X_i were made with a digital survey meter, yielding the values: 12,148, 12,067, 12,207, 12,232, 12,284, 12,129, 11,862, 11,955, 12,044, and 12,150.

The estimated value x_i is the arithmetic mean of the values $X_{i,k}$.

$$x_i = X_i \frac{1}{n}\sum_{k=1}^{n} x_{i,k} = \frac{121078}{10} = 12107.8$$

The standard uncertainty of x_i is

$$u(x_i) = \sqrt{\frac{1}{n(n-1)}\sum_{k=1}^{n}(x_{i,k} - \overline{x}_i)^2} = \sqrt{\frac{1}{10(10-1)}\sum_{k=1}^{10}(x_{i,k} - 12107.8)^2}$$

$$= \sqrt{16628.84} = 128.95$$

There are other Type A methods, but all are based on repeated measurements.

Any evaluation of standard uncertainty that is not a Type A evaluation is a Type B evaluation. Sometimes a Type B evaluation of uncertainty involves making a best guess based on all available information and professional judgment. Despite the reluctance to make this kind of evaluation, it is almost always better to make an informed guess about an uncertainty component than to ignore it completely.

There are many ways to perform Type B evaluations of standard uncertainty. One example of a Type B method is the estimation of counting uncertainty using the square root of the observed counts. If the observed count is N, when the Poisson approximation is used, the standard uncertainty of N may be evaluated as $u(N) = \sqrt{N}$. For example, the standard uncertainty of the first value in Example 1, 12,148, could be estimated as $\sqrt{12148} = 110.218$. When N may be very small or even zero, the equation $u(N) = \sqrt{N+1}$ may be preferable.

Another Type B evaluation of an uncertainty $u(x)$ consists of estimating an upper bound, a, for the magnitude of the error of x based on professional judgment and the best available information. If nothing else is known about the distribution of the measured result, then after a is estimated, the standard uncertainty may be calculated using the equation

$$u(x) = \frac{a}{\sqrt{3}}, \qquad \textbf{(G-10)}$$

which is the standard deviation of a random variable uniformly distributed over the interval $(x - a, x + a)$. The variable a is called the half-width of the interval. Suppose in Example 1, all that was given was the observed range of the data from an analog survey meter dial, i.e., from 11,862 to 12,284, a difference of 422. If it was assumed that the data came from a uniform distribution across this range, then the average is (11,862+12,284)/2 = 12,073, and an estimate of the standard uncertainty would be $u(x) = \frac{211}{\sqrt{3}} = 121.821$.

Given the same information on the range, if values near the middle of the range were considered more likely than those near the endpoints, a triangular distribution may be more appropriate. The mean would be the same as above, 12,073. However the standard uncertainty then be calculated using the equation

$$u(x) = \frac{a}{\sqrt{6}} = \frac{211}{\sqrt{6}} = 86.14 \qquad \textbf{(G-11)}$$

which is the standard deviation of a random variable with a triangular distribution over the interval $(x - a, x + a)$.

When the estimate of an input quantity is taken from an external source, such as a book or a calibration certificate, the stated standard uncertainty can be used.

G.2.1.4 Evaluate the Covariances, $u(x_i,x_j)$, for all Pairs of Input Estimates with Potentially Significant Correlations

A Type A evaluation of the covariance of the input estimates x_i = and x_j = is

$$u(x_i, x_j) = \frac{1}{n(n-1)} \sum_{k=1}^{n} (x_{i,k} - \overline{x_i})(x_{j,k} - \overline{x_j}) \qquad \textbf{(G-12)}$$

An evaluation of variances and covariances of quantities determined by the method of least squares may also be a Type A evaluation. Evaluation of the covariance of two input estimates, x_i and x_j, whose uncertainties are evaluated by Type B methods may require expert judgment. In such cases it may be simpler to estimate the correlation coefficient, $r(x_i,x_j) = [u(x_i,x_j) / u(x_i) \cdot u(x_j)]$, first and then multiply it by the standard uncertainties, $u(x_i)$ and $u(x_j)$ to obtain the covariance, $u(x_i,x_j)$.

A covariance calculation is demonstrated in Example 2 in Section G.2.2.

G.2.1.5 Calculate the Estimate, y, of the Measurand from the Relationship $y = f(x_1,x_2,\ldots,x_N)$

This involves simply substituting, for the particular measurement at hand, the specific values of x_i for the input quantity X_i into the mathematical relationship, $Y = f(X_1,X_2,\ldots,X_N)$, and calculating the result $y = f(x_1,x_2,\ldots,x_N)$.

G.2.1.6 Determine the Combined Standard Uncertainty, $u_c(y)$, of the Estimate, y

The combined standard uncertainty of y is obtained using the following formula:

$$u_c^2(y) = \sum_{i=1}^{N} \left(\frac{\partial f}{\partial x_i} \right)^2 u^2(x_i) + 2 \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \frac{\partial f}{\partial x_i} \frac{\partial f}{\partial x_j} u(x_i, x_j) \qquad \textbf{(G-13)}$$

Here $u^2(x_i)$ denotes the estimated variance of x_i, or the square of its standard uncertainty; $u(x_i,x_j)$ denotes the estimated covariance of x_i and x_j; $\partial f / \partial x_i$ (or $\partial y / \partial x_i$) denotes the partial derivative of f with respect to x_i evaluated at the measured values $x_1,x_2,\ldots,x_N$; and $u_c^2(y)$ denotes the combined variance of y, whose positive square root, $u_c(y)$, is the combined standard uncertainty of y. The partial derivatives, $\partial f / \partial x_i$, are called sensitivity coefficients, usually denoted c_i. The sensitivity coefficient measures how much f changes when x_i changes. Equation G-13 is called the "law of propagation of uncertainty" in the GUM (ISO 1995).

If the input estimates $x_1, x_2, \ldots, x_N$ are uncorrelated, the uncertainty propagation formula reduces to

$$u_c^2(y) = \sum_{i=1}^{N} \left(\frac{\partial f}{\partial x_i} \right)^2 u^2(x_i) \qquad \textbf{(G-14)}$$

Suppose the values $x_1, x_2, \ldots, x_N$ are composed of two groups $w_1, w_2, \ldots, w_n$ and $z_1, z_2, \ldots, z_m$ *with* $N=n+m$. If the w's and the z's are uncorrelated and nonzero, the combined standard uncertainty of $y = \dfrac{w_1 w_2 \ldots w_n}{z_1 z_2 \ldots z_m}$ may be calculated from the formula:

$$u_c^2(y) = y^2 \left(\frac{u^2(w_1)}{w_1^2} + \frac{u^2(w_2)}{w_2^2} + \ldots + \frac{u^2(w_n)}{w_n^2} + \frac{u^2(z_1)}{z_1^2} + \frac{u^2(z_2)}{z_2^2} + \ldots + \frac{u^2(z_m)}{z_m^2} \right) \qquad \textbf{(G-15)}$$

The symbols $z_1, z_2, \ldots, z_m$ have been introduced simply to differentiate those values appearing in the denominator of the model equation from the $w_1, w_2, \ldots, w_n$ appearing in the numerator.

If $y = \dfrac{f(w_1, w_2, \ldots, w_n)}{z_1 z_2 \ldots z_m}$, where f is some specified function of $w_1, w_2, \ldots, w_n$, all the z_i are nonzero, and all the input estimates are uncorrelated. Then:

$$u_c^2(y) = \frac{u_c^2(f(w_1, w_2, \ldots, w_n))}{z_1 z_2 \ldots z_m} + y^2 \left(\frac{u^2(z_1)}{z_1^2} + \frac{u^2(z_2)}{z_2^2} + \ldots + \frac{u^2(z_m)}{z_m^2} \right) \qquad \textbf{(G-16)}$$

An alternative to uncertainty propagation is the use of computerized Monte Carlo methods to propagate not the uncertainties of input estimates but their distributions. Given assumed distributions for the input estimates, the method provides an approximate distribution for the output estimate, from which the combined standard uncertainty or an uncertainty interval may be derived.

G.2.1.7 Optionally Multiply $u_c(y)$ by a Coverage Factor k to Obtain the Expanded Uncertainty U such that the Interval $[y - U, y + U]$ can be Expected to Contain the Value of the Measurand with a Specified Probability

The specified probability, p, is called the level of confidence or the coverage probability and is generally only an approximation of the true probability of coverage. When the distribution of the measured result is approximately normal, the coverage factor is often chosen to be $k = 2$ for a coverage probability of approximately 95%. An expanded uncertainty calculated with $k = 2$ or 3 is sometimes informally called a "two-sigma" or "three-sigma" uncertainty, respectively. The GUM recommends the use of coverage factors in the 2 to 3 range when the combined standard uncertainty represents a good estimate of the true standard deviation. Attachment 19D of MARLAP describes a more general procedure for calculating the coverage factor that gives a desired coverage probability p when there is substantial uncertainty in the value of $u_c(y)$.

G.2.1.8 Report the Result as $y \pm U$ with the Unit of Measurement

At a minimum, state the coverage factor used to compute U and the estimated coverage probability. Alternatively, report the result, y, and its combined standard uncertainty, $u_c(y)$, with the unit of measurement.

The number of significant figures that should be reported for the result of a measurement depends on the uncertainty of the result. A common convention, recommended by MARLAP, is to round the uncertainty (standard uncertainty or expanded uncertainty) to two significant figures and to report both the measured value and the uncertainty to the same number of decimal places. Only final results should be rounded in this manner. Intermediate results in a series of calculation steps should be carried through all steps with additional figures to prevent unnecessary round-off errors. Additional figures are also recommended when the data are stored electronically. Rounding should be performed only when the result is reported.

All results, whether positive, negative, or zero, should be reported as obtained, together with their uncertainties.

A measured value y of a quantity Y that is known to be positive may be so far below zero that it indicates a possible blunder, procedural failure, or other quality control problem. Usually, if

$y + 3u_c(y) < 0$, the result may be invalid. For example, if $y = -10$ and $u_c(y) = 1$, this would imply that Y is negative with high probability, which is known to be impossible. However, if $y = -1$ and $u_c(y) = 1$, the expanded uncertainty covers positive values with reasonable probability. The accuracy of the uncertainty estimate $u_c(y)$ must be considered in evaluating such results, especially in cases where only few counts are observed during the measurement and counting uncertainty is the dominant component of $u_c(y)$. (See MARLAP Chapter 18 and Attachment 19D).

G.2.2 Examples of Some Parameters that Contribute to Uncertainty

The sources of uncertainty described in the following sections, drawn from MARLAP Section 19.5, should be considered.

G.2.2.1 Instrument Background

Single-channel background measurements are usually assumed to follow the Poisson model, in which the uncertainty in the number of counts obtained, N, is given by $\sqrt{N}$. There may be effects that increase the variance beyond what the model predicts. For example, cosmic radiation and other natural sources of instrument background may vary between measurements, the instrument may become contaminated, or the instrument may simply be unstable. Generally, the variance of the observed background is somewhat greater than the Poisson counting variance, although for certain types of instruments, the Poisson model may overestimate the background variance (Currie et al., 1998). If the background does not closely follow the Poisson model, its variance should be estimated by repeated measurements.

The "instrument background," or "instrument blank," is usually measured under the same conditions that will be encountered in the field. Ambient background sources should be minimized, and kept constant during the measurements of M&E. Periodic checks should be made to ensure that the instrument has not picked up additional radioactivity from the M&E during the measurements. If the background drifts or varies nonrandomly over time (i.e., is nonstationary), it is important to minimize the consequences of the drift by performing frequent background measurements.

If repeated measurements demonstrate that the background level is stable, then the average, $\bar{x}$, the results of n similar measurements performed over a period of time may give the best estimate of the background. In this case, if all measurements have the same duration, the experimental standard deviation of the mean, $s\left(\bar{x}\right)$, is also a good estimate of the measurement uncertainty.

Given the Poisson assumption, the best estimate of the uncertainty is still the Poisson estimate, which equals the square root of the summed counts, divided by the number of measurements, $\sqrt{n\bar{x}}\big/n = \sqrt{\bar{x}/n}$ but the experimental standard deviation may be used when the Poisson assumption is invalid. It is always wise to compare the value of $s(\bar{x})$ to the value of the Poisson uncertainty when possible to identify any discrepancies.

G.2.2.2 Counting Efficiency

The counting efficiency for a measurement of radioactivity (usually defined as the detection probability for a particle or photon of interest emitted by the source) may depend on many factors, including source geometry, placement, composition, density, activity, radiation type and energy and other instrument-specific factors. The estimated efficiency is sometimes calculated explicitly as a function of such variables (in gamma-ray spectroscopy, for example). In other cases a single measured value is used (e.g., alpha-particle spectrometry). If an efficiency function is used, the uncertainties of the input estimates, including those for both calibration parameters and sample-specific quantities, must be propagated to obtain the combined standard uncertainty of the estimated efficiency. Calibration parameters tend to be correlated; so, estimated covariances must also be included. If a single value is used instead of a function, the standard uncertainty of the value is determined when the value is measured. An example of the calculation of the uncertainty in counting efficiency is given in Example 2.

Example 2; A radiation counter is calibrated, taking steps to ensure that the geometry of the source position, orientation of the source, pressure, temperature, relative humidity, and other factors that could contribute to uncertainty are controlled, as described below:

The standard source is counted 15 times on the instrument for 300 s.

The radionuclide is long-lived; so, no decay corrections are needed. The uncertainties of the count times are assumed to be negligible.

Within the range of linearity of the instrument, the mathematical model for the calibration is:

$$\varepsilon = \frac{1}{n}\sum_{i=1}^{n}\frac{(N_{S,i}/t_S)-(N_B/t_B)}{a_s} \qquad \textbf{(G-17)}$$

Where:

- ε is the counting efficiency,
- n is the number times the source is counted (15),
- $N_{S,i}$ is the gross count observed during the i^{th} measurement of the source,
- t_S is the source count time (300 s),
- N_{B} is the observed background count (87),
- t_B is the background count time (6,000 s),
- a_S is the activity of the standard source (150.0 Bq). The standard uncertainty of the source, 2.0 Bq, was given by the certificate for the source.

The combined standard uncertainty of ε can be evaluated using Equation G-13. For the purpose of uncertainty evaluation, it is convenient to rewrite the model as:

$$\varepsilon = \frac{\overline{R}}{a_s}$$

Where:

$$\overline{R} = \frac{1}{n}\sum_{i=1}^{n} R_i \quad \text{and} \quad R_i = (N_{S,i}/t_S)-(N_B/t_B)\,, \qquad i = 1,2,\ldots,n$$

The values R_i and their average, $\overline{R}$, are estimates of the count rate produced by the standard, while $\overline{R}/a_{\mathrm{S}}$ is an estimate of the count rate produced by 1 Bq of activity. The standard uncertainty of $\overline{R}$ can be evaluated experimentally from the 15 repeated measurements: $u^2(\overline{R}) = s^2(\overline{R}) = \frac{1}{n(n-1)}\sum_{i=1}^{n}(R_i-\overline{R})^2$. Since only one background measurement was made, the input estimates R_i are correlated with each other. The uncertainty of N_{B}, $u(N_B)=\sqrt{87}$, using a Type B evaluation based on an assumption of a Poisson distribution for the number of background counts.

The covariance between R_i and R_j, for $i \neq j$, may be estimated as

$$u(R_i, R_j) = \frac{\partial R_i}{\partial N_B} \frac{\partial R_j}{\partial N_B} u^2(N_B) = \frac{-1}{t_B} \frac{-1}{t_B} u^2(N_B) = \frac{u^2(N_B)}{t_B^2} = \frac{\sqrt{87}^2}{6000^2} \cong 2 \times 10^{-6}$$

However, the correlation is negligible here because the uncertainty of the background count, N_B, is much smaller than the uncertainty of each source count, $N_{S,i}$. So, the correlation of the input estimates R_i will be approximated as zero (i.e., treated as if they were uncorrelated), and the correlation terms dropped from Equation G-13. This means the evaluation used to calculate the combined standard uncertainty of ε can proceed using equation G-14:

$$u_c^2(y) = \sum_{i=1}^{N} \left(\frac{\partial f}{\partial x_i} \right)^2 u^2(x_i) \text{, so since } \varepsilon = \frac{\overline{R}}{a_s},$$

$$u_c^2(\varepsilon) = \left(\frac{\partial (\frac{\overline{R}}{a_s})}{\partial \overline{R}} \right)^2 u^2(\overline{R}) + \left(\frac{\partial (\frac{\overline{R}}{a_s})}{\partial a_s} \right)^2 u^2(a_s) = \left(\frac{1}{a_s} \right)^2 u^2(\overline{R}) + \left(\frac{-\overline{R}}{a_s^2} \right)^2 u^2(a_s)$$

$$= \left(\frac{u^2(\overline{R})}{a_s^2} \right) + \varepsilon^2 \left(\frac{u^2(a_s)}{a_s^2} \right) \text{. Therefore, } u_c(\varepsilon) = \sqrt{\frac{u^2(\overline{R})}{a_S^2} + \varepsilon^2 \frac{u^2(a_S)}{a_S^2}}$$

Assume the following data were obtained for the 15 separate counts of the calibration source.

Count Number, i	Gross count, $N_{S,i}$	R_i (s^{-1})
1	18,375	61.236
2	18,644	62.132
3	18,954	63.166
4	19,249	64.149
5	19,011	63.356
6	18,936	63.106
7	18,537	61.776
8	18,733	62.429
9	18,812	62.692
10	18,546	61.806

11	18,810	62.686
12	19,273	64.229
13	18,893	62.962
14	18,803	62.662
15	18,280	60.919
	Average, $\overline{R}$ (s^{-1})	62.6202
	Experimental standard deviation, $s(R_i)$ (s^{-1})	0.9483
	Experimental standard deviation of the mean, $s\left(\overline{R}\right)$ (s^{-1})	0.2449

Then the estimated counting efficiency is:

$$\varepsilon = \frac{\overline{R}}{a_s} = \frac{62.6202\ \mathrm{s}^{-1}}{150.0\ \mathrm{Bq}} = 0.4176$$

And the combined standard uncertainty of ε is given by

$$u_c(\varepsilon) = \sqrt{\frac{(0.2449\ \mathrm{s}^{-1})^2}{(150.0\ \mathrm{Bq})^2} + 0.4176^2 \times \frac{(2.0\ \mathrm{Bq})^2}{(150.0\ \mathrm{Bq})^2}} = 0.005802$$

Which may be rounded to 0.0058.

The true counting efficiency may vary because of variations in geometry, position and other influence quantities not explicitly included in the model. These sources of uncertainty may not be controlled as they were in the above example. If this is the case, the standard uncertainty of ε should include not only the standard uncertainty of the estimated mean, as calculated in the example, but also another component of uncertainty due to variations of the true efficiency during subsequent measurements. The additional component may be written as $\varepsilon\phi$, where ϕ is the coefficient of variation (i.e., the standard deviation divided by the mean) of the true efficiency. Then the total uncertainty of ε is obtained by squaring the original uncertainty estimate, adding $\varepsilon^2\phi^2$, and taking the square root of the sum.

$$u_c(\varepsilon) = \sqrt{\frac{u^2(\overline{R})}{a_S^2} + \varepsilon^2\left(\frac{u^2(a_S)}{a_S^2} + \phi^2\right)} \qquad \textbf{(G-18)}$$

In the example above, the experimental variance of the count rates, R_i, may be used to estimate ϕ. Section 18B.2 of Attachment 18B of MARLAP describes an approach for estimating such "excess" variance in a series of measurements.

Variations in counting efficiency due to source placement should be reduced as much as possible through the use of positioning devices that ensure a source with a given geometry is always placed in the same location relative to the detector. If such devices are not used, variations in source position may significantly increase the measurement uncertainty.

Calibrating an instrument under conditions different from the conditions under which M&E sources are counted may lead to large uncertainties in the activity measurements. Source geometry in particular tends to be an important factor for many types of radiation counters. If correction factors are used, their uncertainties should be evaluated and propagated, as mentioned in section G.2.1.1.

G.2.2.3 Digital Displays and Rounding

If a measuring device has a digital display with readability[1] δ, the standard uncertainty of a measured value is at least $\delta / 2\sqrt{3}$, which is the variance of a random variable uniformly distributed over the interval $(x - \delta/2, x + \delta/2)$. Note that this is the same result as given by equation G-10 with $a = \delta/2$. This uncertainty component exists even if the instrument is completely stable.

A similar Type B method may be used to evaluate the standard uncertainty due to computer round-off error. When a value x is rounded to the nearest multiple of 10^n, where n is an integer, the component of uncertainty generated by round-off error is $10^n / (2\sqrt{3})$. This component of uncertainty should be kept small in comparison to the total uncertainty of x by performing

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[1] Readability is the smallest difference that can still be read on a display. For instruments with an analog indicating device, the readability is equal to the smallest fraction of a scale interval that can still be estimated with reasonable reliability or which can be determined by an auxiliary device. For instruments with a numeric indicator (digital display), the readability is equal to one digital step.

rounding properly and printing with an adequate number of figures. In a long calculation involving mixed operations, carry as many digits as possible through the entire set of calculations and then round the final result appropriately as described in MARLAP Section 19.3.7 (MARLAP 2004).

Example 3: The readability of a digital survey doserate meter is 1 nGy/h. Therefore, the minimum standard uncertainty of a measured absorbed dose rate is $1/2\sqrt{3}$ = 0.29 nGy/h.

Example 4: Suppose the results for R_i in Example 2 had been rounded to the nearest whole number before the analysis. Then the average would be computed as 62.6 instead of 62.6202 and the standard deviation would be computed as 0.9103 instead of 0.9483. This demonstrates the effect that rounding intermediate results can have on subsequent calculations. If this rounding to the nearest positive integer had already occurred prior to receiving the data, and the original data were no longer available, a correction for it could be made when estimating the combined standard uncertainty of R_i. The component of uncertainty generated by round-off error is $1/(2\sqrt{3})$:

$$u(R_i) = \sqrt{0.9103^2 + \left(\frac{1}{2\sqrt{3}}\right)^2} = 0.9549.$$

G.2.3 Example Uncertainty Calculation

To illustrate how the uncertainty calculations are performed in practice, the following example is given based on that of Lewis et al. (Lewis 2005). The calculation will be that of the combined standard uncertainty in the calibration of a surface contamination monitor.

G.2.3.1 Model Equation and Sensitivity Coefficients

Surface contamination monitors are calibrated in terms of their response to known rates of radioactive emissions. In practice this is achieved by using large-area, planar sources that have a defined area and whose emission rates have been determined in a traceable manner. The calibration is usually determined in terms of response per emission rate per unit area. In this example, the source is positioned with its active face parallel to and at a distance of 3 mm from the face of the detector. The monitor detector area (50 cm^2) is smaller than the area of the calibration source, which is a 10 cm × 10 cm layer of ^{14}C on a thick aluminum substrate. The monitor has an analog display and has a means to set the detector voltage.

The efficiency, ε, is defined by:

$$\varepsilon = \frac{(M - B) \times f_V \times f_d \times f_u \times f_{bs}}{\left(E/A \right)} \tag{G-19}$$

Where:

M	observed monitor reading, s^{-1}
B	background reading, s^{-1}
E	emission rate of the calibration source, s^{-1}
A	area of the active portion of the calibration source, cm^2
f_V	plateau voltage factor,
f_d	source-detector separation factor,
f_u	source uniformity factor,
f_{bs}	backscatter factor.

The sensitivity coefficients of Equation G-19 are given by:

$$\frac{\partial \varepsilon}{\partial M} = (A/E) \times f_V \times f_d \times f_u \times f_{bs} = \frac{\varepsilon}{(M - B)} \tag{G-20}$$

$$\frac{\partial \varepsilon}{\partial B} = -(A/E) \times f_V \times f_d \times f_u \times f_{bs} = \frac{-\varepsilon}{(M - B)} \tag{G-21}$$

$$\frac{\partial \varepsilon}{\partial E} = -(M - B)(A/E^2) \times f_V \times f_d \times f_u \times f_{bs} = \frac{-\varepsilon}{E} \tag{G-22}$$

$$\frac{\partial \varepsilon}{\partial A} = (M - B)(1/E) \times f_V \times f_d \times f_u \times f_{bs} = \frac{\varepsilon}{A} \tag{G-23}$$

$$\frac{\partial \varepsilon}{\partial f_V} = (M - B)(A / E) \times f_d \times f_u \times f_{bs} = \frac{\varepsilon}{f_V} \qquad \textbf{(G-24)}$$

$$\frac{\partial \varepsilon}{\partial f_d} = (M - B)(A / E) \times f_V \times f_u \times f_{bs} = \frac{\varepsilon}{f_d} \qquad \textbf{(G-25)}$$

$$\frac{\partial \varepsilon}{\partial f_u} = (M - B)(A / E) \times f_V \times f_d \times f_{bs} = \frac{\varepsilon}{f_u} \qquad \textbf{(G-26)}$$

$$\frac{\partial \varepsilon}{\partial f_{bs}} = (M - B)(A / E) \times f_V \times f_d \times f_u = \frac{\varepsilon}{f_{bs}} \qquad \textbf{(G-27)}$$

Under normal conditions, the factors f_V, f_d, f_u and f_{bs} are each assumed to have a value of one. If the uncertainties are to be calculated in relative terms, the uncertainty equation becomes (see Equation G-16):

$$\left(\frac{\sigma_C}{\varepsilon}\right)^2 = \left(\frac{M}{M-B}\right)^2 \left(\frac{\sigma_M}{M}\right)^2 + \left(\frac{B}{M-B}\right)^2 \left(\frac{\sigma_B}{B}\right)^2 + \left(\frac{\sigma_E}{E}\right)^2 + \left(\frac{\sigma_A}{A}\right)^2 + \left(\frac{\sigma_{f_V}}{f_V}\right)^2 + \left(\frac{\sigma_{f_d}}{f_d}\right)^2 + \left(\frac{\sigma_{f_u}}{f_u}\right)^2 + \left(\frac{\sigma_{f_{bs}}}{f_{bs}}\right)^2 \textbf{(G-28)}$$

If the relative uncertainties are all expressed as percentages, $\left(\frac{\sigma_{x_i}}{x_i}\right)$, where x_i is an input quantity, then the combined standard uncertainty will be a percentage. The relative sensitivity coefficients, c_i, are the terms multiplying each relative uncertainty term $\left(\frac{\sigma_{x_i}}{x_i}\right)$ in Equation G-28. This approach produces relative sensitivity coefficients of unity for the last 6 terms.

G.2.3.2 Uncertainty Components

<u>Monitor reading of source, *M* (Type A)</u>

Several techniques can be used to determine the mean observed monitor reading, *M*, and its uncertainty. Assume a snap-shot technique is used whereby six successive, but randomly timed, readings are recorded, giving 350, 400, 400, 325, 350, 350 s^{-1}. The mean and standard deviation of the mean becomes 362.5 ± 12.5 s^{-1}. This equates to a percentage uncertainty in *M* of 3.45% and the relative sensitivity coefficient from Equation G-28, $\frac{M}{(M-B)}$, is 362.5/(362.5 – 32.5), which is equal to 1.10. The distribution is assumed to be normal.

<u>Monitor reading of background, *B* (Type B)</u>

In this case, an eye-averaging technique was used whereby the highest and lowest count rates were recorded over a given period of time. These count rates were 40 and 25 s^{-1} respectively, giving a mean value of 32.5 s^{-1}. This value is assumed to have a rectangular distribution with a half-width of 7.5 s^{-1}, and an uncertainty of $7.5/\sqrt{3} = 4.330$, equating to a percentage uncertainty of $4.330/32.5 = 0.1332$ or 13%. The relative sensitivity coefficient from Equation G-28, $\dfrac{B}{(M-B)}$, is 32.5/(362.5 - 32.5), which gives a value of 0.098.

<u>Emission rate of calibration source, *E* (Type B)</u>

The emission rate of the source and its uncertainty were provided on the calibration certificate by the laboratory that calibrated the source using a windowless proportional counter. The statement on the certificate was:

"The measured value of the emission rate is $E = 2{,}732 \pm 13$ s^{-1}

The reported uncertainty is based on a standard uncertainty multiplied by a coverage factor of $k = 2$, which provides a level of confidence of approximately 95%. The standard uncertainty on E is therefore 13/2 = 6.5 s^{-1} or 0.24%. Unless the certificate provides information to the contrary, it is assumed that the uncertainty has a normal distribution.

<u>Source area, *A* (Type B)</u>

In the absence of an uncertainty statement by the manufacturer, the only information available is the product drawing that shows the active area dimensions to be 10 cm × 10 cm. On the assumption that the outer bounds of the length, L, and the width, W, are 9.9 and 10.1 cm, the uncertainty of the linear dimensions may be taken to be a rectangular distribution with a half-width of 0.1 cm.

$L = 10$ and $u(L) = 0.1/\sqrt{3} = 0.0577$. $W = 10$ and $u(W) = 0.1/\sqrt{3} = 0.0577$. Since $A = LW$, we get $u^2(A) = u^2(LW) = L^2u^2(W) + W^2u^2(L) = 2(10)^2(0.0577)^2 = 0.665858$, therefore $u(A) = 0.816\,\text{cm}^2$ or 0.816%.

Plateau voltage factor, f_V (Type B)

This applies only to those instruments where voltage adjustments are possible. If the setting is not checked and/or adjusted between calibrations, then this has no effect. Changing the plateau voltage without performing a recalibration is not recommended. If, however, the user is allowed to do this, the setting may not be returned to exactly that used during the calibration. In this particular example, the slope of the response curve in this region is taken to be 10% / 50 v. It is assumed that an operator is more likely to set the voltage nearer to the optimum than the extremes and that ± 50 v represents the range at the 100% confidence level. Accordingly, a triangular distribution is assumed with a half-width of 50 v, equating to an uncertainty for the voltage of $50/\sqrt{6} = 20.4124$ and an uncertainty for the voltage factor of 20.4124(10%)/ 50 = 4.0825%.

Source-detector separation factor, f_d (Type B)

This effect arises from the uncertainty in mounting the calibration source exactly 3 mm from the detector face. Experimental evidence has shown that, for the particular ^{14}C source at 3 mm source-detector separation, the change in response was 2.6% / mm. It is assumed that the deviation from the nominal 3 mm separation is no greater than 1 mm but that all values are equally probable between 2 and 4 mm, a rectangular distribution. The uncertainty in the separation is thus $1/\sqrt{3} = 0.5774$. The uncertainty of the separation factor is thus 0.5774 mm × 2.6% / mm, equal to 1.5011%.

Non-uniformity of calibration source, f_u (Type B)

Large area sources may have a non-uniform activity distribution across their surfaces. For the ^{14}C source, the uniformity is assumed to be better than ± 10%. This is based on comparing 10 cm^2 sections of the source. For a typical monitor with a detector area of 50 cm^2 and a calibration source area of 100 cm^2, a worst-case condition could be that the area under the detector has an activity per unit area that is 10% greater than the mean value for the whole source. (The outer area correspondingly will be 10% less than mean value.) Assuming a rectangular distribution, this represents an uncertainty of $10/\sqrt{3} = 5.774\%$ for the source non-uniformity factor.

<u>Backscatter factor, f_{bs} (Type B)</u>

Variations in backscatter effects arise from factors such as the nature of the surface on which the calibration source is resting and the proximity to scattering surfaces such as walls. This effect can be quite marked for photon emitters, but for ^{14}C on aluminum substrates the effect is negligible.

G.2.3.3 Uncertainty Budget

An important part of the uncertainty analysis is to determine which factors are contributing the most to the overall uncertainty.

Table G.3: Uncertainty Budget for the Efficiency Example

Source of uncertainty	Type	Probability distribution	*Relative Sensitivity Coeffient,* c_i	$u_i(x_i)$ (%)	$u_i(y)=c_i\,u_i(x_i)$ (%)	$(u_i(y))^2$	$(u_i(y))^2/Total$
Standard deviation of mean of M	A	Normal	1.10	3.45	3.80	14.44	0.21
Standard deviation of mean of B	B	Rectangular	0.098	13.32	1.31	1.72	0.02
Standard uncertainty of calibration source emission rate, E	B	Normal	1.0	0.24	0.24	0.06	0.00
Half -width of source length, L and width W on the area A	B	Product of 2 independent rectangular	1.0	0.816	0.816	0.666	0.01
Half -width of voltage factor, f_V	B	Triangular	1.0	4.08	4.08	16.65	0.24
Half -width of source-detector separation factor, f_d	B	Rectangular	1.0	1.50	1.50	2.25	0.03
Half-width of calibration source non-uniformity factor, f_u	B	Rectangular	1.0	5.77	5.77	33.29	0.48
Uncertainty of backscatter factor, f_{bs}	B	n.a.	1.0	0.0	0.0	0.00	0.00
Combined standard uncertainty		Normal	- - -	- - -	8.31 = $\sqrt{69.07}$	Total= 69.07	0.99
Expanded uncertainty (k=2)		Normal	- - -	- - -	2·8.31= 16.6	- - -	

The relative sensitivity coefficients, c_i, are the terms multiplying each relative uncertainty term $\left(\frac{\sigma_{x_i}}{x_i}\right)$ in Equation G-28. To do this, each component of uncertainty $u_i(y)=c_i\, u_i(x_i)$ is squared to give its component of variance $(u_i(y))^2$. These are totaled to get the total variance, in this case 69.07. Finally, the ratio of each component of variance to the total is computed.

Examining the last column of the uncertainty budget table (Table G.3) shows that the major source of uncertainty is due to source non-uniformity (48%) followed by the voltage factor (24%) and the reading of the source (21%). Thus, to decrease the overall uncertainty, attention should be paid to those factors first.

G.2.3.4 Reported Result

Using the formula above, the calibration factor in terms of emission rate becomes:

$$\varepsilon = \frac{(M-B)\times f_V \times f_d \times f_u \times f_{bs}}{\left(E/A\right)} = \frac{(362.5-32.5)\times 1\times 1\times 1\times 1}{\left(2732/100\right)} = 12.1\ (\text{counts}\times\text{s}^{-1})/(\text{s}^{-1}\times\text{cm}^{-2})$$

The combined standard uncertainty is (12.1)(.0831) = 1.0056. The reported expanded uncertainty will be 2.0, based on a standard uncertainty of 1.0 multiplied by a coverage factor of $k = 2$, which provides a level of confidence of approximately 95%.

G.3 Calculation of the Minimum Detectable Concentration

Table G.4 Notation for Section G.3

Symbol	***Definition***	***Formula or reference***	***Type***
ε	efficiency		
F	calibration function	$X = F(Y)$	
F^{-1}	evaluation function	$Y = F^{-1}(X)$, closely related to the *mathematical model* $Y = f(X_1, X_2, \ldots, X_N)$	
S_C.	Critical net signal	Net signal is calculated from the gross signal by subtracting the estimated blank value and any interferences	
S_D	Mean value of the net signal that gives a specified probability, 1−β, of yielding an observed signal greater than its critical value S_C.		
X	observable response variable, measureable signal		
x_C.	The critical value of the response variable	Calculation of y_C requires the choice of a significance level for the test. The significance level is a specified upper bound for the probability, α, of a Type I error. The significance level is usually chosen to be 0.05.	If a measured value exceeds the critical value, a decision is made that radiation or radioactivity has been detected
Y	state variable, measurand		
y_C	Critical value of the concentration	$y_C = F^{-1}(x_C)$.	
$y_D = \frac{S_D}{\varepsilon}$	Minimum detectable concentration (MDC)	$y_D = \frac{S_D}{\varepsilon}$	

G.3.1 Critical Value

In the terminology of ISO 11843-1 (1997), the measured concentration is the state variable, denoted by Y, which represents the state of the material being analyzed. The state variable usually cannot be observed directly, but it is related to an observable response variable, denoted by X, through a calibration function F, the mathematical relationship being written as $X = F(Y)$. The response variable X is most often an instrument signal, such as the number of counts observed. The inverse, $Y = F^{-1}(X)$ of the calibration function is sometimes called the evaluation function. The evaluation function, which gives the value of the net concentration in terms of the response variable, is closely related to the mathematical model $Y = f(X_1, X_2, \ldots, X_N)$ described in Section G.2.1.1.

In a Scenario B detection decision, either the null or alternative hypothesis is chosen on the basis of the observed value of the response variable, X. The value of X must exceed a certain threshold value to justify rejection of the null hypothesis and acceptance of the alternative hypothesis. This threshold is called the critical value of the response variable and is denoted by x_C.

The calculation of x_C requires the choice of a significance level for the test. The significance level is a specified upper bound for the probability, α, of a Type I error. The significance level is usually chosen to be 0.05. This means that when there is no radiation or radioactivity present (above background), there should be at most a 5% probability of incorrectly deciding that it is present.

The critical value of the concentration, y_C, is defined as the value obtained by applying the evaluation function, F^{-1}, to the critical value of the response variable, x_C. Thus, $y_C = F^{-1}(x_C)$. When x is the gross instrument signal, this formula typically involves subtraction of the background signal and division by the counting efficiency, and possibly other factors.

A detection decision can be made by comparing the observed gross instrument signal to its critical value, x_C, as indicated above. However, it has become standard practice to make the decision by comparing the net instrument signal to its critical value, S_C. The net signal is calculated from the gross signal by subtracting the estimated blank value and any interferences.[2] The critical net signal, S_C, is calculated from the critical gross signal, x_C, by subtracting the same correction terms; so, in principle, either approach should lead to the same detection decision.

Since the term "critical value" alone is ambiguous, one should specify the variable to which the term refers. For example, one may discuss the critical (value of the) radionuclide concentration, the critical (value of the) net signal, or the critical (value of the) gross signal. In this document, the signal is usually a count, and the critical value generally refers to the net count.

The response variable is typically an instrument signal, whose mean value generally is positive even when there is radioactivity present (i.e., above background). The gross signal must be

[2] Interference is the presence of other radiation or radioactivity that hinder the ability to analyze for the radiation or radioactivity of interest.

corrected by subtracting an estimate of the signal produced under those conditions. See Section G.2.2.1 (Instrument Background).

G.3.2 Minimum Detectable Concentration

The minimum detectable concentration (MDC) is the minimum concentration of radiation or radioactivity that must be present in a sample to give a specified power, $1 - \beta$. It may also be defined as:

- The minimum radiation or radioactivity concentration that must be present to give a specified probability, $1 - \beta$, of detecting the radiation or radioactivity; or
- The minimum radiation or radioactivity concentration that must be present to give a specified probability, $1 - \beta$, of measuring a response greater than the critical value, leading one to conclude correctly that there is radiation or radioactivity present.

The *power* of any hypothesis test is defined as the probability that the test will reject the null hypothesis when it is false, i.e., the correct decision. Therefore, if the probability of a Type II error is denoted by β, the power is $1 - \beta$. In the context of radiation or radioactivity detection, the power of the test is the probability of correctly detecting the radiation or radioactivity (concluding that the radiation or radioactivity is present), which happens whenever the response variable exceeds its critical value. The power depends on the concentration of the radiation or radioactivity and other conditions of measurement; so, one often speaks of the "power function" or "power curve." Note that the power of a test for radiation or radioactivity detection generally is an increasing function of the radiation or radioactivity concentration − i.e., the greater the radiation or radioactivity concentration the higher the probability of detecting it.

In the context of MDC calculations, the value of β that appears in the definition, like α, is usually chosen to be 0.05 or is assumed to be 0.05 by default if no value is specified. The minimum detectable concentration is denoted in mathematical expressions by y_D. The MDC is usually obtained from the minimum detectable value of the net instrument signal, S_D. S_D, is defined as the mean value of the net signal that gives a specified probability, $1 - \beta$, of yielding an observed signal greater than its critical value S_C. The relationship between the critical net signal, S_C, and the minimum detectable net signal, S_D, is shown in Figure 5.2 in Section 5.7.2.

The term MDC must be carefully and precisely defined to prevent confusion. The MDC is by definition an estimate of the true concentration of the radiation or radioactivity required to give a specified high probability that the measured response will be greater than the critical value.

The common practice of comparing a measured concentration to the MDC, instead of to the S_C, to make a detection decision is incorrect. If this procedure were used, then there would be only a a 50% chance of deciding that radioactivity was present when the concentration was actually at the MDC. This is in direct contradiction to the definition of MDC. See MARLAP Appendix B, Attachment B1 for a further discussion of this issue.

Since the MDC is calculated from measured values of input quantities such as the counting efficiency and background level, the MDC estimate has a combined standard uncertainty, which in principle can be obtained by uncertainty propagation. To avoid confusion, it may be useful to remember that a detection decision is usually made by comparing the instrument response to the critical value, and that the critical value generally does not even have the units of radiation or radioactivity concentration.

G.3.3 Calculation of the Critical Value

If the net signal is a count, then in many circumstances the uncertainty in the count can be estimated by a Type B evaluation using the fact that for a Poisson distribution with mean N_B, the variance is also N_B. Thus the uncertainty in the background count is estimated as $\sqrt{N_B}$.

Hence, the critical value is often an expression involving $\sqrt{N_B}$.

The most commonly used approach for calculating the critical net signal, $S_{C,}$ is given by the following equation.[3]

[3] This expression for the critical net count depends for its validity on the assumption of Poisson counting statistics. If the variance of the blank signal is affected by interferences, or background instability, then Equation 20.7 of MARLAP may be more appropriate.

$$S_C = z_{1-\alpha}\sqrt{N_B \frac{t_S}{t_B}\left(1+\frac{t_S}{t_B}\right)} \qquad \textbf{(G-29)}$$

Where:

N_B is the background count,

t_S is the count time for the sample,

t_B is the count time for the background, and

$z_{1-\alpha}$ is the $(1-\alpha)$-quantile of the standard normal distribution.

Example 5: A 6,000-second background measurement is performed on a proportional counter and 108 beta counts are observed. A sample is to be counted for 3,000 s. Estimate the critical value of the net count when α = 0.05.

$$S_C = z_{1-\alpha}\sqrt{N_B \frac{t_S}{t_B}\left(1+\frac{t_S}{t_B}\right)}$$

$$S_C = 1.645\sqrt{108\times\left(\frac{3{,}000\ \text{s}}{6{,}000\ \text{s}}\right)\left(1+\frac{3{,}000\ \text{s}}{6{,}000\ \text{s}}\right)} = 14.8 \text{ net counts}$$

If α = 0.05 and $t_B = t_S$, equation G-29 leads to the well-known expression $2.33\sqrt{N_B}$ for the critical net count (Currie, 1968).

When the background count is high (e.g., 100 or more) Equation G-29 works well, but at lower background levels it can produce a high rate of Type I errors. Since this is a Scenario B hypothesis test, this means that too often a decision will be made that there is radiation or radioactivity present when it actually is not.

When the mean background counts are low and $t_B \neq t_S$, another approximation formula for S_C appears to out-perform all of the other approximations reviewed in MARLAP, namely the Stapleton Approximation:

$$S_C = d \times \left(\frac{t_S}{t_B} - 1 \right) + \frac{z_{1-\alpha}^2}{4} \times \left(1 + \frac{t_S}{t_B} \right) + z_{1-\alpha} \sqrt{\left(N_B + d \right) \frac{t_S}{t_B} \left(1 + \frac{t_S}{t_B} \right)} \qquad \textbf{(G-30)}$$

When $\alpha = 0.05$, setting the parameter $d = 0.4$ yields the best results. When, in addition, $t_B = t_S$, the Stapleton approximation gives the equation

$$S_C = 1.35 + 2.33\sqrt{N_B + 0.4} \qquad \textbf{(G-31)}$$

G.3.4 Calculation of the Minimum Detectable Value of the Net Instrument Signal

The traditional method for calculating the MDC involves two steps: first calculating the minimum detectable value of the net instrument signal and then converting the result to a concentration using the mathematical measurement model.

The minimum detectable value of the net instrument signal, denoted by S_D, is defined as the mean value of the net signal that gives a specified probability, $1 - \beta$, of yielding an observed signal greater than its critical value S_C.

The MDC may be estimated by calculating the minimum detectable value of the net instrument signal, S_D, and converting the result to a concentration.

Counting data rarely, if ever, follow the Poisson model exactly, but the model can be used to calculate S_D if the variance of the background signal is approximately Poisson and a conservative value of the efficiency constant, ε, is used to convert S_D to y_D. The equation below shows how to calculate S_D using the Poisson model.

$$S_D = S_C + \frac{z_{1-\beta}^2}{2} + z_{1-\beta} \sqrt{\frac{z_{1-\beta}^2}{4} + S_C + R_B t_S \left(1 + \frac{t_S}{t_B} \right)} \qquad \textbf{(G-33)}$$

Where:

S_C is the critical value,

R_B is the mean count rate of the blank, $R_B = \dfrac{N_B}{t_B}$,

N_B is the background count,

t_S is the count time for the test source,

t_B is the count time for the background, and

$z_{1-\beta}$ is the $(1-\beta)$-quantile of the standard normal distribution.

When Equation G-29 is appropriate for the critical net count, and $\alpha = \beta$, this expression for S_D simplifies to $z_{1-\beta}^2 + 2S_C$. If in addition, $\alpha = \beta = 0.05$ and $t_B = t_S$ then

$$S_D = 2.71 + 2S_C = 2.71 + 2(2.33\sqrt{N_B}) = 2.71 + 4.66\sqrt{N_B}$$

Example 6 A 6,000-second background measurement on a proportional counter produces 108 beta counts and a source is to be counted for 3,000 s. Assume the background measurement gives the available estimate of the true mean background count rate, R_B and use the value 0.05 for Type I and Type II error probabilities. From Section,G.3.3 Example 5, the critical net count, S_C, equals 14.8, so $S_D = z_{1-\beta}^2 + 2S_C = 1.645^2 + 2\,(14.8) = 32.3$ net counts.

When the Stapleton approximation (Equation G-30) is used for S_C, the minimum detectable net count S_D may be calculated using the equation G-33, but when the Poisson model is assumed, a better estimate is given by the equation:

$$S_D = \frac{(z_{1-\alpha} + z_{1-\beta})^2}{4}\left(1 + \frac{t_S}{t_B}\right) + (z_{1-\alpha} + z_{1-\beta})\sqrt{R_B t_S \left(1 + \frac{t_S}{t_B}\right)} \quad \textbf{G-34)}$$

This equation is the same as that recommended by ISO 11929-1 (ISO 2000) in a slightly different form.

When $\alpha = \beta = 0.05$ and $t_\mathrm{B} = t_\mathrm{S}$, the preceding equation becomes:

$$S_D = 5.41 + 4.65\sqrt{R_B t_S} \quad \textbf{(G-35)}$$

Consult MARLAP Chapter 20 for a discussion of the calculation of S_D and y_D when both Poisson counting statistics and other sources of variance are considered.

G.3.5 Calculation of the Minimum Detectable Concentration

The MDC is often used to compare different measurement procedures against specified requirements. The calculation of the nominal MDC is complicated by the fact that some input quantities in the mathematical model, such as interferences, counting efficiency, and instrument background may vary significantly from measurement to measurement. Because of these variable quantities, determining the value of the radiation or radioactivity concentration that corresponds to the minimum detectable value of the net instrument signal, S_D, may be difficult in practice. One common approach to this problem is to make conservative choices for the values of the variable quantities, which tend to increase the value of the MDC.

The mean net signal, S, is usually directly proportional to Y, the true radiation or radioactivity concentration present. Hence, there is a efficiency constant, ε, such that $S = \varepsilon Y$. The constant ε is typically the mean value of the product of factors such as the source count time, decay-correction factor, and counting efficiency. Therefore, the value of the minimum detectable concentration, y_D, is

$$y_D = \frac{S_D}{\varepsilon} \quad \textbf{(G-36)}$$

The preceding equation is only true if all sources of variability are accounted for when determining the distribution of the net signal, $\hat{S}$. Note that ensuring the MDC is not underestimated also requires that the value of ε not be overestimated.

Using any of the equations in Section G.3.4 to calculate S_D is only appropriate if a conservative value of the efficiency constant, ε, is used when converting S_D to the MDC.

Example 7: Consider a scenario where $t_B = 6{,}000$ s, $t_S = 3{,}000$ s, and $R_B \approx 0.018\ \text{s}^{-1}$. Let the measurement model be $Y = \dfrac{N_S - (N_B t_S / t_B)}{t_S \varepsilon}$

Where:

Y is the activity of the radionuclide in the sample and

ε is the counting efficiency (counts per second)/(Bq/cm^2)

Assume the source count time, t_S, has negligible variability, the counting efficiency has mean 0.42 and a 10% relative combined standard uncertainty, and from Example 6, S_D = 32.3 net counts.

The mean minimum detectable concentration is $y_D = \dfrac{S_D}{t_S \varepsilon} = \dfrac{32.3}{(3000)(0.42)} = 0.0256\ \text{Bq/cm}^2$.

Adjusting for the 10% variability in the counting efficiency, the uncertainty is (0.10) × (0.42) = 0.042. Assuming that the efficiency is normally distributed, the lower 5th percentile for ε is (0.42) - (1.645)(0.042) = 0.35, where -1.645 is the 5th percentile of a standard normal distribution.. Therefore a conservative estimate of the efficiency constant is ε = 0.35 and a conservative estimate of the minimum detectable concentration is:

$$y_D = \frac{S_D}{t_S \varepsilon} = \frac{32.3}{(3000)(0.35)} = 0.0308\ \text{Bq/cm}^2.$$

An alternative procedure could be to recognize that because of the uncertainties in the input estimates entered into the measurement model to convert from S_D to Y, that the MDC is actually a random variable. Then the methods for propagation of uncertainty given in Section G.2 can be applied. Using the same assumptions as above we would find that y_D = 0.0256 ± 0.0051 with 95% confidence based on a coverage factor of 2. Therefore the 95% upper confidence level for y_D would be 0.0307 Bq.

More conservative (higher) estimates of the MDC may be obtained by following NRC recommendations (NRC 1984), in which formulas for the MDC include estimated bounds for relative systematic error in the background determination (Δ_B) and the sensitivity (Δ_A). The critical net count S_C is increased by $\Delta_B N_B \dfrac{t_S}{t_B}$, and the minimum detectable net count S_D is increased by $2\Delta_B\ N_B \dfrac{t_S}{t_B}$. Next, the MDC is calculated by dividing S_D by the efficiency and multiplying the result by $1+\Delta_A$. The conservative approach presented in NRC 1984 treats random errors and systematic errors differently to ensure that the MDC for a measurement process is unlikely to be consistently underestimated, which is an important consideration if it is required by regulation or contract to achieve a specified MDC.

G.4 Calculation of the Minimum Quantifiable Concentration

Table G.5 Notation for Section G.4

Symbol	***Definition***	***Formula or reference***	***Type***
k_Q	Multiple of the standard deviation defining y_Q, usually chosen to be 10.	$k_Q = \dfrac{\sqrt{\sigma^2(y \mid Y = y_Q)}}{y_Q}$	Chosen during DQO process
$\sigma^2(y \mid Y = y_Q)$	The variance of the estimator y given the true concentration Y equals y_Q.		Theoretical
y_Q	Minimum quantifiable concentration (MQC)	The concentration at which the measurement process gives results with a specified relative standard deviation $1/k_Q$, where k_Q is usually chosen to be 10.	Theoretical

Calculation of the MQC requires that one be able to estimate the standard deviation for the result of a hypothetical measurement performed on a sample with a specified radionuclide concentration. The MQC is defined symbolically as the value y_Q that satisfies the relation:

$$y_Q = k_Q \sqrt{\sigma^2(y \mid Y = y_Q)} \qquad \textbf{(G-37)}$$

Where the specified relative standard deviation of y_Q is $1/k_Q$ (usually chosen to be 10% so that $k_Q = 10$). $\sigma^2(y \mid Y = y_Q)$ is the variance of the estimator y given the true concentration Y equals y_Q. If the function $\sigma^2(y \mid Y = y_Q)$ has a simple form, it may be possible to solve the above equation for y_Q using only algebraic manipulation. Otherwise, fixed-point iteration, or other more general approaches, may be used, as discussed in MARLAP Section 20.4.3.

When Poisson counting statistics are assumed, and the mathematical model for the radionuclide concentration is $Y = S/\varepsilon$, where S is the net count, S/t_S is the net count rate and ε is the efficiency of the measurement, the above equation may be solved for y_Q to obtain:

$$y_Q = \frac{k_Q^2}{2t_S\varepsilon(1-k_Q^2\phi_{\hat{\varepsilon}}^2)}\left(1+\sqrt{1+\frac{4(1-k_Q^2\phi_{\hat{\varepsilon}}^2)}{k_Q^2}\left(R_B t_S\left(1+\frac{t_S}{t_B}\right)+R_I t_S+\sigma^2(\widehat{R}_I)t_S^2\right)}\right) \qquad \textbf{(G-38)}$$

Where:

t_S	is the count time for the source, s,
t_B	is the count time for the background, s,
R_B	is the mean background count rate, s^{-1},
R_I	is the mean interference count rate, s^{-1},
$\sigma(\hat{R}_I)$	is the standard deviation of the measured interference count rate, s^{-1}, and
$\phi_{\hat{\varepsilon}}^2$	is the relative variance of the measured efficiency, $\hat{\varepsilon}$.

If the efficiency ε may vary, then a conservative value, such as the 0.05-quantile $\varepsilon_{0.05}$, should be substituted for ε in the formula. Note that $\phi_{\hat{\varepsilon}}^2$ denotes only the relative variance of $\hat{\varepsilon}$ due to subsampling and measurement error – it does not include any variance of the efficiency ε itself (see discussion in Section G.2).

Note that equation G-38 defines the MQC only if $1 - k_Q^2\phi_{\hat{\varepsilon}}^2 > 0$. If $1 - k_Q^2\phi_{\hat{\varepsilon}}^2 \leq 0$, the MQC is infinite, because there is no concentration at which the relative standard deviation of y fails to exceed $1 / k_Q$. In particular, if the relative standard deviation of the measured efficiency $\hat{\varepsilon}$ exceeds $1 / k_Q$, then $1 - k_Q^2\phi_{\hat{\varepsilon}}^2 < 0$ and the MQC is infinite.

If there are no interferences, equation G-37 simplifies to:

$$y_Q = \frac{k_Q^2}{2t_S\varepsilon(1 - k_Q^2\phi_{\hat{\varepsilon}}^2)}\left(1 + \sqrt{1 + \frac{4(1 - k_Q^2\phi_{\hat{\varepsilon}}^2)}{k_Q^2}\left(R_B t_S\left(1 + \frac{t_S}{t_B}\right)\right)}\right) \tag{G-39}$$

Example 8: Consider the scenario of Example 5, where t_B = 6,000 s, t_S = 3,000 s, and $R_B \approx 0.018\ s^{-1}$. Suppose the measurement model is $Y = \dfrac{N_S - (N_B t_S / t_B)}{t_S\varepsilon}$

Where:

Y is the specific activity of the radionuclide in the sample and

ε the counting efficiency (counts per second)/(Bq/cm^2).

Assume:

The source count time, t_S, has negligible variability,

the counting efficiency has mean 0.42 and a 5% relative combined standard uncertainty, and

S_D = 32.3 net counts. S_D / t_S =32.3/3000 is the net count rate.

The counting efficiency ε = 0.42

The mean minimum detectable concentration is $y_D = \frac{S_D}{t_S \varepsilon} = \frac{32.3}{(3000)(0.42)} = 0.0256 \text{ Bq/cm}^2$.

Also assume:

$k_Q = 10$

$\phi_{\hat{\varepsilon}} = 0.05$

$\phi_{\hat{\varepsilon}}^2 = 0.05^2$

$1 - k_Q^2 \phi_{\hat{\varepsilon}}^2 = 1 - 100 \times (0.05^2) = 0.75$, and

there are no interferences so that equation G-38 can be used.

Note that if the counting efficiency has mean 0.42 and a 10% relative standard uncertainty as in Example 11, then $1 - k_Q^2 \phi_{\hat{\varepsilon}}^2 = 1 - 100 \times (0.10^2) = 0$ and the MQC would be infinite. Therefore it was necessary to change the procedure for evaluating the efficiency in this example so that the relative combined standard uncertainty could be reduced. In this example it is assumed to be 5%.

The MQC can be calculated as:

$$y_Q = \frac{k_Q^2}{2 t_S \varepsilon (1 - k_Q^2 \phi_{\hat{\varepsilon}}^2)} \left(1 + \sqrt{1 + \frac{4(1 - k_Q^2 \phi_{\hat{\varepsilon}}^2)}{k_Q^2} \left(R_B t_S \left(1 + \frac{t_S}{t_B} \right) + 0 \right)} \right)$$

$$y_Q = \frac{100}{2\,(3000)(0.42)(0.75)} \left(1 + \sqrt{1 + \frac{4\,(0.75)}{100} \left((0.018 \text{ s}^{-1})(3000 \text{ s}) \left(1 + \frac{(3000 \text{ s})}{(6000 \text{ s})} \right) + 0 \right)} \right)$$

$= 0.151\ \text{Bq/cm}^2$

As a check, y_Q can be calculated in a different way. If y_Q is the MQC and $k_Q = 10$, then the relative combined standard uncertainty of a measurement of concentration y_Q is 10%. The procedure described in Section 5.6 can be used to predict the combined standard uncertainty of a measurement made on a hypothetical sample whose concentration is exactly $y_Q = 0.151\ \text{Bq/cm}^2$.

The measurement model is $Y = \dfrac{N_S - (N_B t_S / t_B)}{t_S \varepsilon}$.

Recall from Section G.2.1.6 that if $y = \dfrac{f(x_1, x_2, \ldots, x_n)}{z_1 z_2 \ldots z_m}$, where f is some specified function of $x_1, x_2, \ldots, x_n$, all the z_i are nonzero, and all the input estimates are uncorrelated that the combined standard uncertainty may be calculated using Equation G-16:

$$u_c^2(y) = \frac{u_c^2(f(x_1, x_2, \ldots, x_n))}{z_1 z_2 \ldots z_m} + y^2 \left(\frac{u^2(z_1)}{z_1^2} + \frac{u^2(z_2)}{z_2^2} + \ldots + \frac{u^2(z_m)}{z_m^2} \right)$$

Substituting

$y = Y$

$f(x_1, x_2, \ldots, x_n) = f(N_S, N_B, t_S, t_B) = N_S - (N_B t_S / t_B) / t_S$

$z_1 = \varepsilon$, and

$$u_c^2(N_S - (N_B t_S / t_B) / t_S) = u_c^2(N_S / t_S) + u_c^2((N_B t_S / t_B) / t_S) = \frac{u_c^2(N_S) + (t_S / t_B)^2 u_c^2(N_B)}{t_S^2} =$$

$$\frac{\sqrt{N_S}^2 + \sqrt{N_B}^2 (t_S^2 / t_B^2)}{t_S^2} = \frac{N_S + N_B (t_S^2 / t_B^2)}{t_S^2}$$

Results in:

$$u_c^2(Y) = \frac{N_S + (N_B t_S^2 / t_B^2)}{t_S^2 \varepsilon^2} + Y^2 \left(\frac{u^2(\varepsilon)}{\varepsilon^2} \right) \text{ or}$$

$$u_c(Y) = \sqrt{\frac{N_S + (N_B t_S^2 / t_B^2)}{t_S^2 \varepsilon^2} + Y^2 \left(\frac{u^2(\varepsilon)}{\varepsilon^2} \right)}$$

Inserting the values

$Y = y_Q = 0.151$ Bq/cm^2

$t_B = 6{,}000$ s

$t_S = 3{,}000$ s

$\varepsilon = 0.42$ (counts per second)/(Bq/cm^2)

$N_B = R_B t_B = (0.018\ \text{s}^{-1})(3{,}000\ \text{s}) = 108$ and

$N_S = x_Q t_S \varepsilon + R_B t_B = (0.151\ \text{Bq})(3000\ \text{s})(0.42) + (0.018\ \text{s}^{-1})(3{,}000\ \text{s}) = 244.26$

yields

$$u_c(Y) = \sqrt{\frac{244.26 + (108)(3{,}000)^2 / (6{,}000)^2}{(3000)^2 (0.42)^2} + (0.151)^2 (0.05^2)} = 0.0151\ \text{Bq/cm}^2$$

Thus, the uncertainty at $y_Q = 0.151$ is 0.0151 and the relative uncertainty is 0.1, so y_Q is verified to be the MQC.

As in example 7, we adjust for the (now) 5% relative combined standard uncertainty in the counting efficiency. The uncertainty is (0.05) × (0.42) = 0.02142. Assuming that the efficiency is normally distributed, the lower 5th percentile is (0.42) - (1.645)(0.021) = 0.385. Therefore a conservative estimate of the efficiency is ε = 0.385 and a conservative estimate of the minimum detectable concentration is: $y_Q = \frac{(0.151)(0.42)}{0.385} = 0.165\ \text{Bq/cm}^2$.